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B.Tech. DEGREE EXAMINATION, DECEMBER 2015
First Semester

15MA101 – CALCULUS AND SOLID GEOMETRY
(For the candidates admitted during the academic year 2015 – 2016)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

1. The Eigen values of the matrix $\begin{bmatrix} 1 & 2 \\ 5 & 4 \end{bmatrix}$ are
   (A) 1, 6 (B) −1, 6
   (C) 1, −6 (D) −1, −6

2. If $A = \begin{bmatrix} 3 & 5 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{bmatrix}$, then the Eigen values of A^{-1} are
   (A) $1, \frac{1}{3}, \frac{1}{4}$ (B) 1, 3, 4
   (C) $1^2, 3^2, 4^2$ (D) $1, \frac{1}{3^2}, \frac{1}{4^2}$

3. Two Eigen values of $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ are 3 and 6, then the third Eigen value is
   (A) 1 (B) 2
   (C) 3 (D) 4

4. The index of the canonical form $-y_1^2 + y_2^2 + 4y_3^2$ is
   (A) 3 (B) 2
   (C) 1 (D) 0

5. If $f(x,y) = 0$ and y is an implicit function of x, then $\frac{dy}{dx}$ is
   (A) $\frac{-\partial f / \partial x}{\partial f / \partial y}$ (B) $\frac{\partial f / \partial y}{\partial f / \partial x}$
   (C) $\frac{-\partial f / \partial y}{\partial f / \partial x}$ (D) $\frac{\partial f / \partial x}{\partial f / \partial y}$

6. If $u = x^2-y^2$, $v = 2xy$, then the value of $\frac{\partial(u,v)}{\partial(x,y)}$ is

(A) $4(x^2-y^2)$ (B) x^2+y^2

(C) $4(x^2+y^2)$ (D) x^2-y^2

7. The point at which there is no extreme value is

(A) Maximum point (B) Minimum point

(C) Saddle point (D) Stationary point

8. If u is a homogeneous function of degree 'n' then by Euler's theorem, we have

(A) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=nu$ (B) $x\frac{\partial u}{\partial x}-y\frac{\partial u}{\partial y}=nu$

(C) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=(n-1)u$ (D) $x\frac{\partial u}{\partial x}-y\frac{\partial u}{\partial y}=(n-1)u$

9. The complimentary function of $(D^2-2D+1)y = 0$ is

(A) $C_1e^x+C_2e^{-x}$ (B) $(C_1+C_2x)e^x$

(C) $C_1e^{2x}+C_2e^{-2x}$ (D) $(C_1+C_2x)e^{-x}$

10. The roots of the auxiliary equation of $m^2-4 = 0$ are

(A) ± 2 (B) $\pm 2i$

(C) $\pm\sqrt{2}$ (D) $1\pm 2i$

11. If $1\pm 2i$ are the roots of a differential equation $f(D)y = 0$, then the complementary function is

(A) Ae^x+Be^{-2x} (B) $e^{-2x}(A\cos x-B\sin x)$

(C) $e^x(A\cos 2x+B\sin 2x)$ (D) $e^{-x}(A\cos 2x+B\sin 2x)$

12. The particular integral of $(D^2+16)y=e^{-4x}$ is

(A) $\frac{x}{32}e^{-4x}$ (B) $\frac{1}{32}e^{-4x}$

(C) $\frac{1}{16}e^{-4x}$ (D) $\frac{x}{16}e^{-4x}$

13. The curvature of a circle of radius 'r' is

(A) r (B) $\frac{1}{r}$

(C) $\frac{1}{r^2}$ (D) r^2

14. _____ is defined as the locus of centre of curvature.

(A) Involute (B) Evolute

(C) Radius of curvature (D) Envelope

15. The radius of curvature of the curve $y = 4\sin x$ at $x=\frac{\pi}{2}$ is

(A) 1/4 (B) −1/4

(C) 1/2 (D) −1 /2

16. The curvature of the straight line is
(A) 1 (B) 2
(C) 0 (D) –1

17. The radius of the sphere $x^2+y^2+z^2-2y-4z = 11$ is
(A) – 4 (B) 4
(C) 3 (D) –3

18. Two spheres cut each other orthogonally, if the tangent planes at a point of intersection
(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{6}$
(C) $\frac{\pi}{3}$ (D) 2π

19. The centre of the sphere $x^2+y^2+z^2-2x+4y-4z = 0$ is
(A) (–1, 2, –2) (B) (–2, 4, –4)
(C) (2, –4, 4) (D) (1, –2, 2)

20. The section of a right circular cone by any plane perpendicular to its axis.
(A) Cone (B) Circle
(C) Sphere (D) Cylinder

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. Two of the Eigen values of $A=\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ are 3 and 6. Find the Eigen values of A^{-1} and A^2.

22. Find the Taylor series expansion for x^y at (1, 1) upto second degree terms.

23. Solve: $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+4y=e^{-2x}$.

24. Find radius of curvature at $\left(\frac{1}{4},\frac{1}{4}\right)$ on the curve $\sqrt{x}+\sqrt{y}=1$.

25. Find the envelope of the straight lines $y = mx + a/m$ where m is a parameter and 'a' is constant.

26. Find $\frac{du}{dt}$ of $u=\cosh\frac{x}{y}$ and $x=t^2$, $y=e^t$.

27. Solve $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}=0$.

PART – C (5 × 12 = 60 Marks)

Answer **ALL** Questions

28. a. Verify Cayley-Hamilton theorem for $A=\begin{bmatrix}1 & 2 & 3\\ 2 & -1 & 4\\ 3 & 1 & -1\end{bmatrix}$ and hence find A^{-1} and A^4.

(OR)

b. Reduce the quadratic form $Q=3x_1^2+5x_2^2+3x_3^2-2x_1x_2-2x_2x_3+2x_3x_1$ to a diagonal Cannonical form and hence find its rank, index and signature.

29. a. Find the extreme values of $sinx+siny+sin(x+y)$.

(OR)

b. Find the volume of the largest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

30. a. Solve $(x+2)^2\frac{d^2y}{dx^2}-(x+2)\frac{dy}{dx}+y=3x+4$.

(OR)

b. Solve using variation of parameter method $\frac{d^2y}{dx^2}+4y=4\tan x$.

31. a. Find the equation of circle of curvature of the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ at $\left(\frac{a}{4},\frac{a}{4}\right)$.

(OR)

b. Find the evolute of the curve $x = acos^3\theta$, $y = asin^3\theta$.

32.a.i Find the equation of the sphere having the circle $x^2+y^2+z^2-2x+4y-6z+7=0$ and $2x-y+2z = 5$ for a great circle.

ii. Show that the sphere $x^2+y^2-z^2 = -9$ and $x^2+y^2+z^2-6x+13y-2z+9 = 0$ cut orthogonally.

(OR)

b. i. Find the equation of the right circular cone whose vertex is at (2, –3, 5) axis makes equal angles with coordinate axis and the semi vertical angle is measured to be 30°.

ii. Find the equation of the right circular cylinder of radius 3 and axis $\frac{x-1}{2}=\frac{y-3}{2}=\frac{z-5}{-1}$.

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